

Subject (L.O 3)

Date

Ex:- Eliminate x, y from the eqn

$$ax^2 + 2hxy + by^2 + 2fx + 2gy + C = 0$$

$$a(x'\cos\alpha - y'\sin\alpha)^2 + 2h(x'\cos\alpha - y'\sin\alpha)(x'\sin\alpha + y'\cos\alpha) + b(x'\sin\alpha + y'\cos\alpha)^2 + 2f(x'\cos\alpha - y'\sin\alpha) + 2g(x'\sin\alpha + y'\cos\alpha) + C = 0$$

$$C.O \ x' y' = 0$$

$$-a\sin 2\alpha + 2h\cos 2\alpha + b\sin 2\alpha = 0$$

$$(a-b)\sin 2\alpha = 2h\cos 2\alpha$$

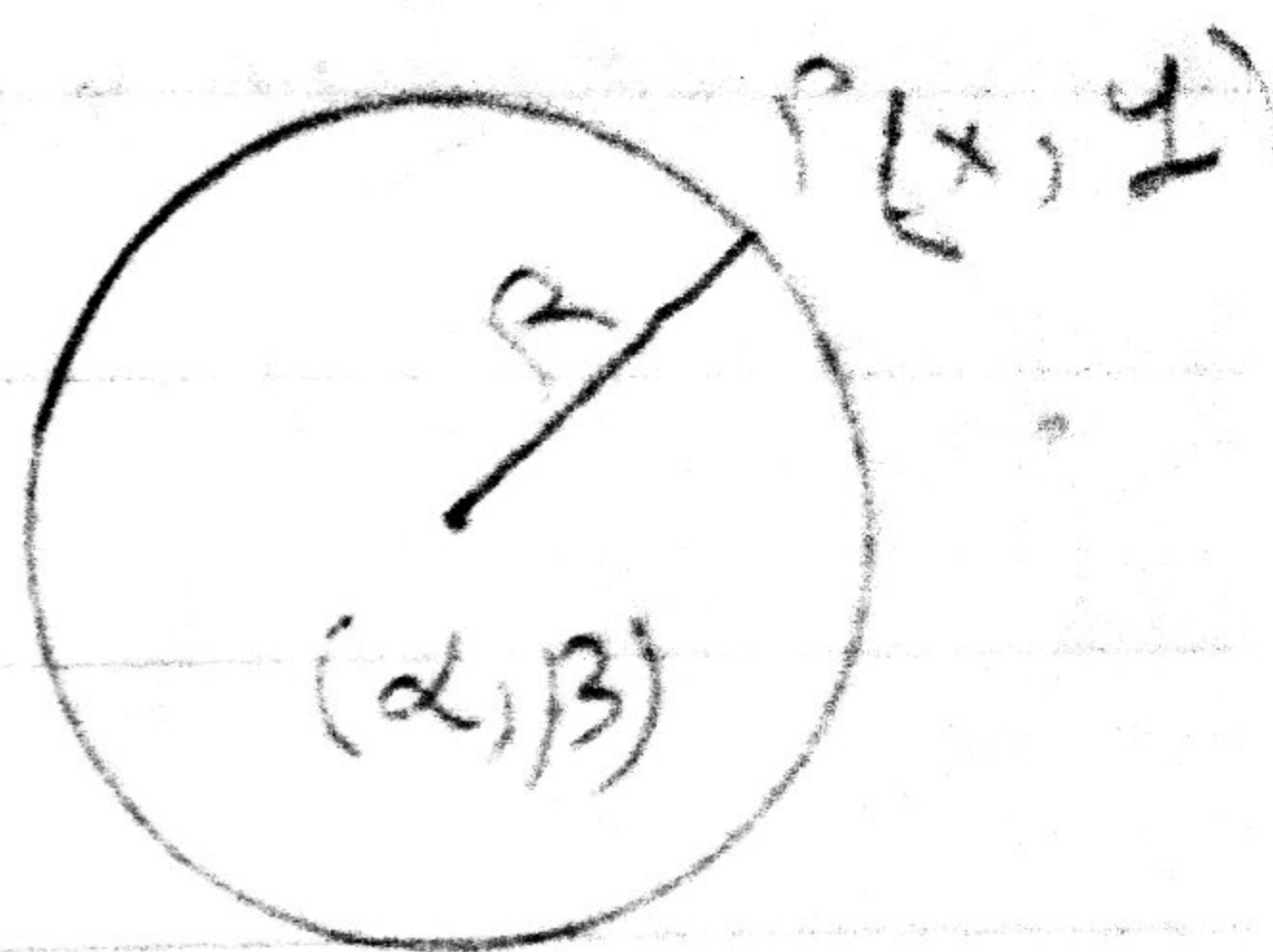
$$\tan 2\alpha = \frac{2h}{a-b}$$

The circle:-

* A circle thr the center (α, β) and radius R

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$$(x - \alpha)^2 + (y - \beta)^2 = R^2$$



- If $(\alpha, \beta) = (0, 0) \Rightarrow x^2 + y^2 = R^2$

The General Form of the circle is:-

$$x^2 + y^2 + 2fx + 2gy + c = 0$$

⊛ ⊛ $\Rightarrow$ circle

if $a = b, h = 0$

$$(x + f)^2 - f^2 + (y + g)^2 - g^2 + c = 0$$

$$(x + f)^2 + (y + g)^2 = f^2 + g^2 - c$$

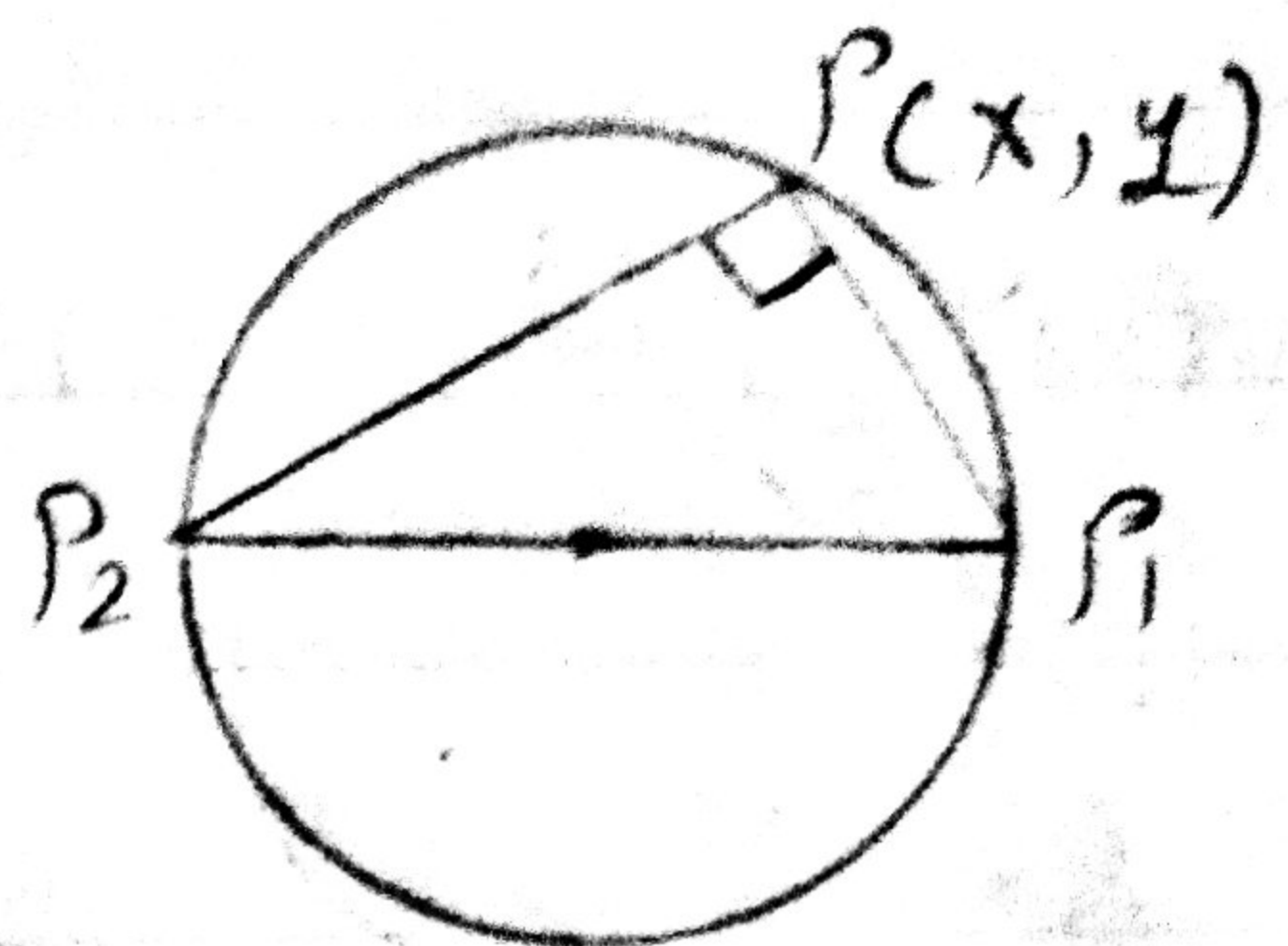
* Center is $(-f, -g)$

* $R = \sqrt{f^2 + g^2 - c}$

provided $f^2 + g^2 - c \geq 0$

★ A circle thr two pts P_1, P_2 which are the ends of a diameter

$$(X - X_1)(X - X_2) + (Y - Y_1)(Y - Y_2) = 0 \quad \#$$

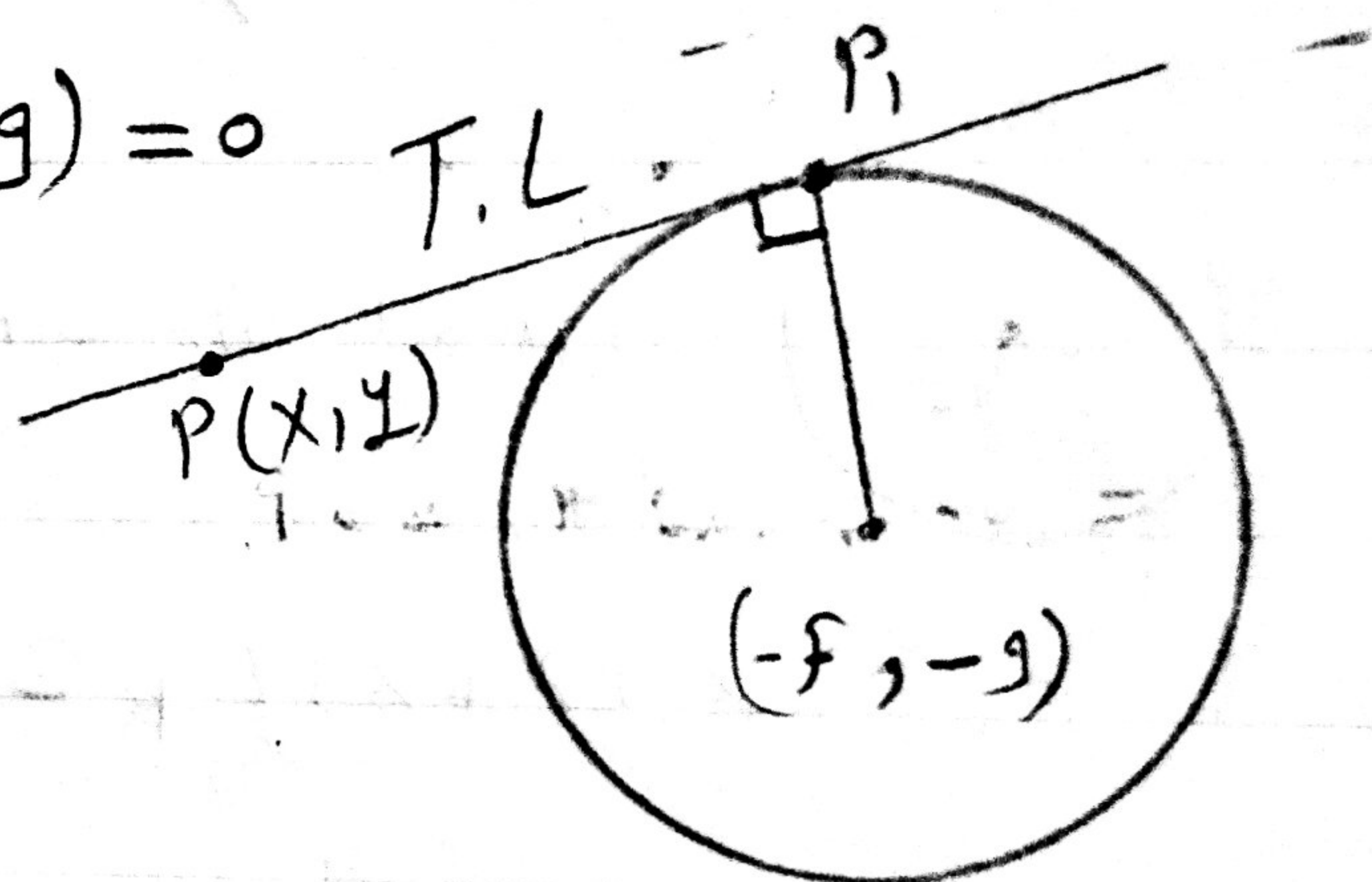


Tangent line eqn (T.L) to (d) from P_1 on (d).

$$(X - X_1)(X_1 + F) + (Y - Y_1)(Y_1 + G) = 0$$

$$XX_1 + XF - X_1^2 - X_1F$$

$$YY_1 + YG - Y_1^2 - Y_1G = 0$$



$$X_1^2 + Y_1^2 + 2FX_1 + 2GY_1 + C = 0$$

$$2FX_1 + 2GY_1 + C = -X_1^2 - Y_1^2$$

$$XX_1 + YY_1 + F(X + X_1) + G(Y + Y_1) + C = 0$$

Ex :- Find the T.L to the circle $x^2 + y^2 - 4x + 8y + 11 = 0$
from $P_1(2, 7)$

P_1 on (d)

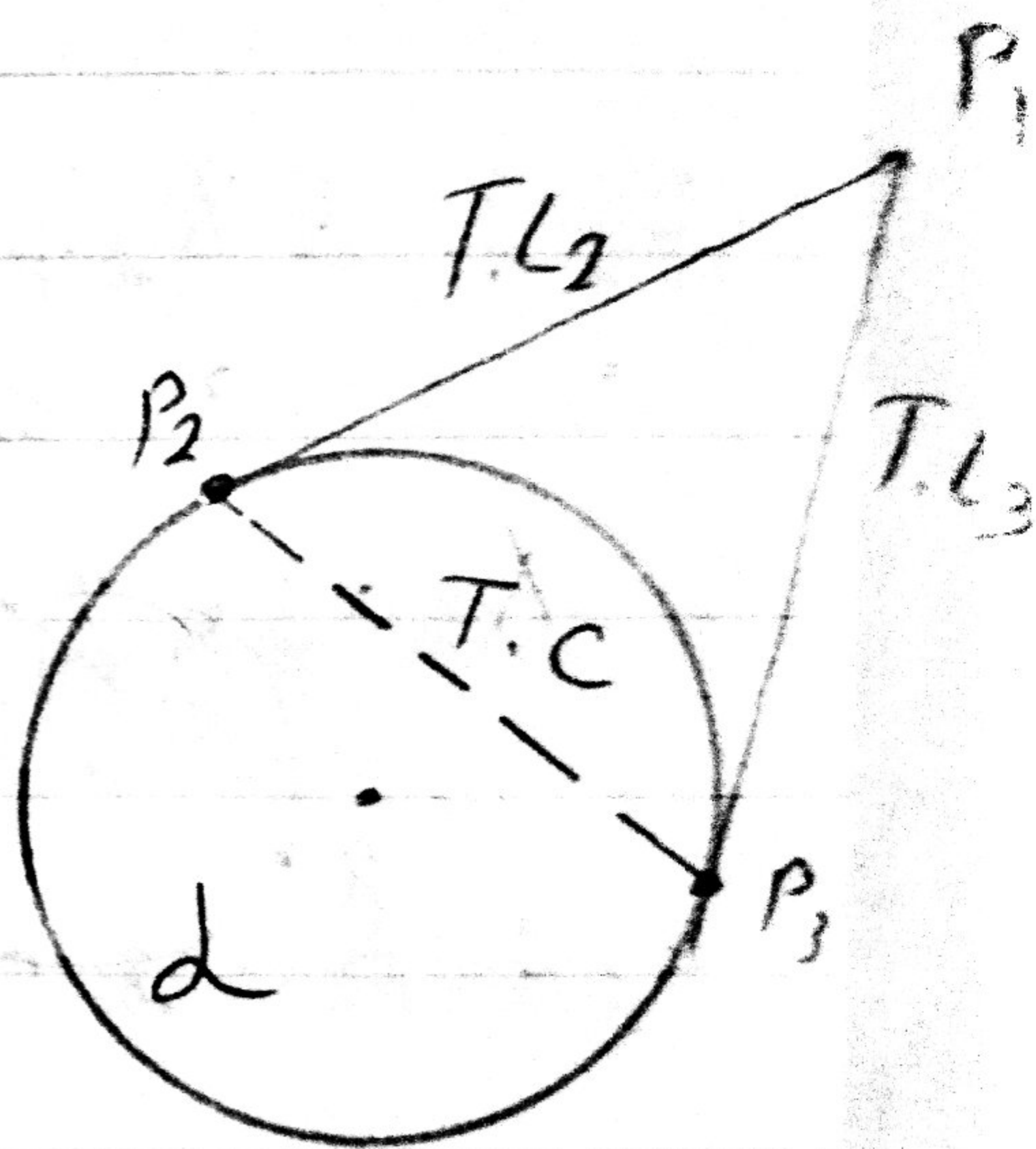
T.L is : $2x + 7y - 2(x+2) - 4(y+7) + 11 = 0$

* Tangential chord to (d) from P_1 :-

T.L₂ : $xx_2 + yy_2 + f(x+x_2) + g(y+y_2) + c = 0$

تقع P_1 على T.L₂

$\therefore x_1x_2 + y_1y_2 + f(x_1+x_2) + g(y_1+y_2) + c = 0 \rightarrow 1)$



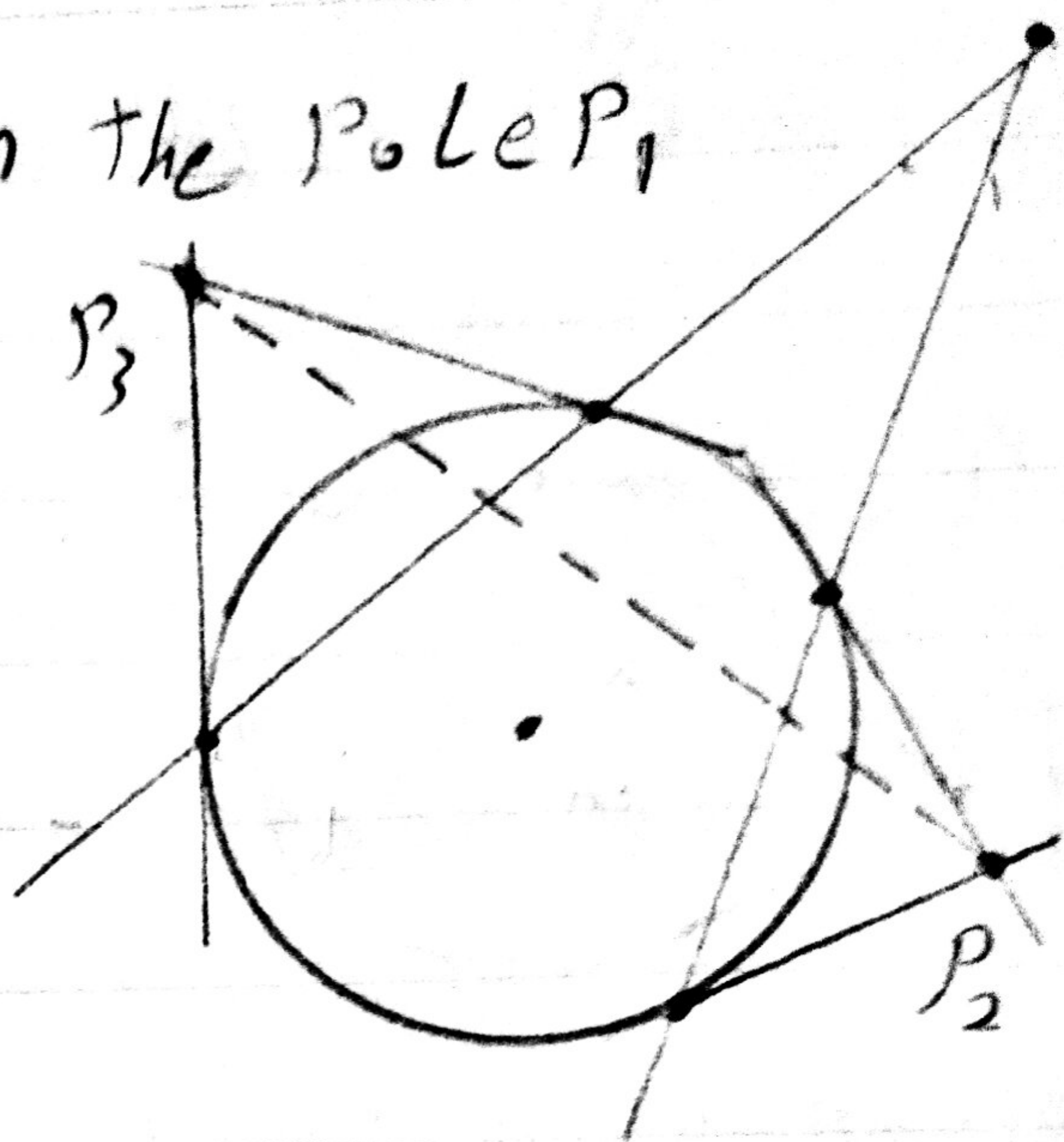
T.L₃ : $xx_3 + yy_3 + f(x+x_3) + g(y+y_3) + c = 0$

تقع P_1 على T.L₃

$\therefore x_1x_3 + y_1y_3 + f(x_1+x_3) + g(y_1+y_3) + c = 0 \rightarrow 2)$

T.C : $xx_1 + yy_1 + f(x+x_1) + g(y+y_1) + c = 0$

polar line (P.L) to (d) From the Pole P_1



$$T.C_2: X X_2 + Y Y_2 + F(X + X_2) + g(Y + Y_2) + C = 0$$

P_1 تقع على

$$\therefore X_1 X_2 + Y_1 Y_2 + F(X_1 + X_2) + g(Y_1 + Y_2) + C = 0 \rightarrow //$$

$$T.C_3: X X_3 + Y Y_3 + F(X + X_3) + g(Y + Y_3) + C = 0$$

P_1 تقع على

$$\therefore X_1 X_3 + Y_1 Y_3 + F(X_1 + X_3) + g(Y_1 + Y_3) + C = 0$$

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$$P.L: X X_1 + Y Y_1 + F(X + X_1) + g(Y + Y_1) + C = 0$$

Ex: Find The pole of the polar line

$$2x + y + 12 = 0 \rightarrow$$

$$\text{to } 2: x^2 + y^2 - 4x + 3y - 1 = 0$$

Let the pole to be $P_1(x_1, y_1)$

P.L from P_1 to (2) is:

$$xx_1 + yy_1 - 2(x + x_1) + \frac{3}{2}(y + y_1) - 1 = 0$$

$$(x_1 - 2)x + (y_1 + \frac{3}{2})y + (-2x_1 + \frac{3}{2}y_1 - 1) = 0 \rightarrow (2)$$

$$(1) \equiv (2)$$

$$\frac{x_1 - 2}{2} = \frac{y_1 + \frac{3}{2}}{1} = \frac{-2x_1 + \frac{3}{2}y_1 - 1}{12} = k$$

$$x_1 = 2k + 2, \quad y_1 = k - \frac{3}{2}$$

$$-2(2k + 2) + \frac{3}{2}(k - \frac{3}{2}) - 1 = 12k$$

$$\therefore k = -\frac{1}{2}$$

$$x_1 = 1, \quad y_1 = -2$$

$$P_1(1, -2)$$